

Critical values of the Chi-square distribution with d degrees of freedom

d	Probability of exceeding the critical value			d	Probability of exceeding the critical value		
	0.05	0.01	0.001		0.05	0.01	0.001
1	3.841	6.635	10.828	11	19.675	24.725	31.264
2	5.991	9.210	13.816	12	21.026	26.217	32.910
3	7.815	11.345	16.266	13	22.362	27.688	34.528
4	9.488	13.277	18.467	14	23.685	29.141	36.123
5	11.070	15.086	20.515	15	24.996	30.578	37.697
6	12.592	16.812	22.458	16	26.296	32.000	39.252
7	14.067	18.475	24.322	17	27.587	33.409	40.790
8	15.507	20.090	26.125	18	28.869	34.805	42.312
9	16.919	21.666	27.877	19	30.144	36.191	43.820
10	18.307	23.209	29.588	20	31.410	37.566	45.315

INTRODUCTION TO POPULATION GENETICS, Table D.1
© 2013 Sinauer Associates, Inc.

The Chi-Square test of independence is used to determine if there is a significant relationship between two nominal (categorical with no numerical value, like gender or relatedness) variables. The frequency of each category for one nominal variable is compared across the categories of the second nominal variable. The data can be displayed in a contingency table where each row represents a category for one variable and each column represents a category for the other variable. For example, say a researcher wants to examine the relationship between gender (male vs. female) and empathy (high vs. low). The chi-square test of independence can be used to examine this relationship. The null hypothesis for this test is that there is no relationship between gender and empathy. The alternative hypothesis is that there is a relationship between gender and empathy (e.g. there are more high-empathy females than high-empathy males).

For example, let's suppose a drug trial is conducted on a group of patients and it is hypothesized that the patients receiving the drug would show increased heart rates compared to those that did not receive the drug. The results of the test could then be analyzed to determine whether the difference in heart rates is considered significant, and degrees of freedom are part of the calculations.

Because degrees of freedom calculations identify how many values in the final calculation are allowed to vary, they can contribute to the validity of an outcome. These calculations are dependent upon the sample size, or observations, and the parameters to be estimated, but generally, in statistics, degrees of freedom equal the number of observations minus the number of parameters. This means there are more degrees of freedom with a larger sample size.

Formula for Degrees of Freedom

The statistical formula to determine degrees of freedom is quite simple. It states that degrees of freedom equal the number of values in a data set minus 1, and looks like this:

$$df = N - 1$$

Where **N** is the number of values in the data set (sample size). Take a look at the sample computation.

If there is a data set of 4, ($N=4$).

Call the data set X and create a list with the values for each data.

For this example data, set X includes: 15, 30, 25, 10

This data set has a **mean**, or average of 20. Calculate the mean by adding the values and dividing by N:

$$(15+30+25+10)/4 = 20$$

Using the formula, the degrees of freedom would be calculated as $df = N - 1$:

$$\text{In this example, it looks like, } df = 4 - 1 = 3$$

This indicates that, in this data set, three numbers have the freedom to vary as long as the mean remains 20.

Note: Null Hypothesis = the hypothesis that there is no significant difference between specified populations so any observed difference is due to sampling/experimental error.

Degrees of freedom	Not Significant						α	Significant			
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005	
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597	
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838	
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860	
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750	
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548	
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278	
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955	
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589	
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188	
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757	
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299	
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819	
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319	
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801	
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267	
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718	
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156	
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582	
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997	
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401	
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796	
23	9.262	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181	
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559	
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928	
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290	
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645	
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993	
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336	
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672	
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766	
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490	
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952	
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215	
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321	
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299	
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169	

Not Significant → must accept the null hypothesis which states that there is no significant difference between specified populations so any observed difference is due to sampling/experimental error.

Significant → must reject the null hypothesis, meaning the results of the experiment differ from the expected outcomes only by chance.